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Problem: how can we define a switching law σ such that the solution of the switched system converge to x_r?

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Fixed $x_r \in \mathbb{R}^2$, the objective is, given an initial condition $x_0 \in \mathbb{R}^2$, to define a switching law σ such that

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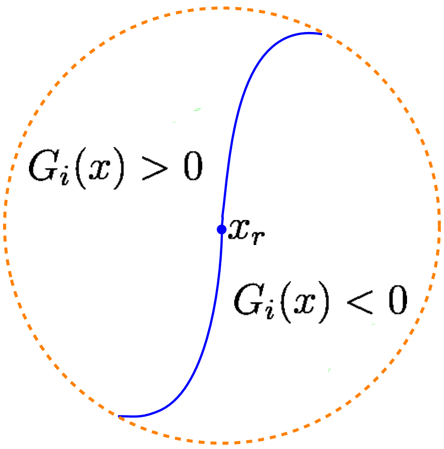

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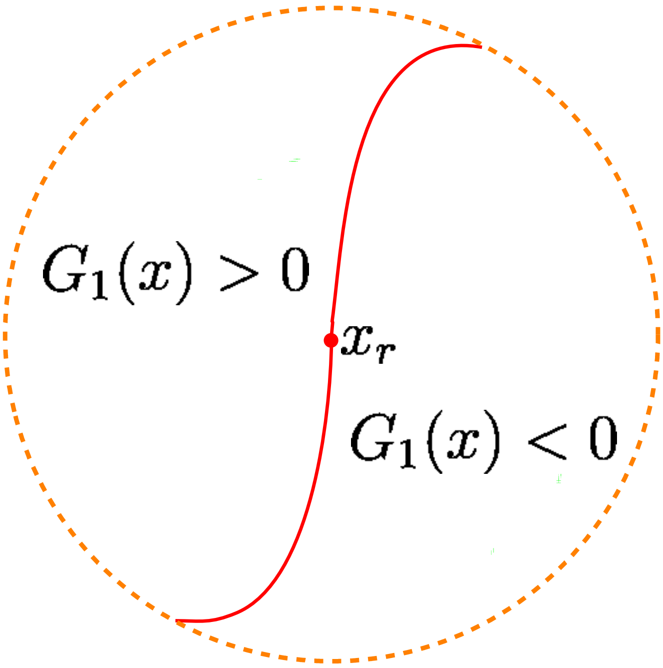

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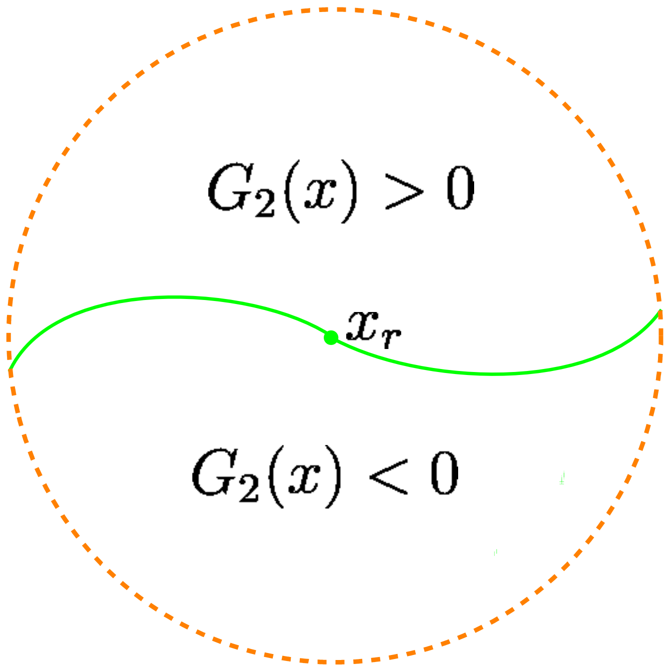

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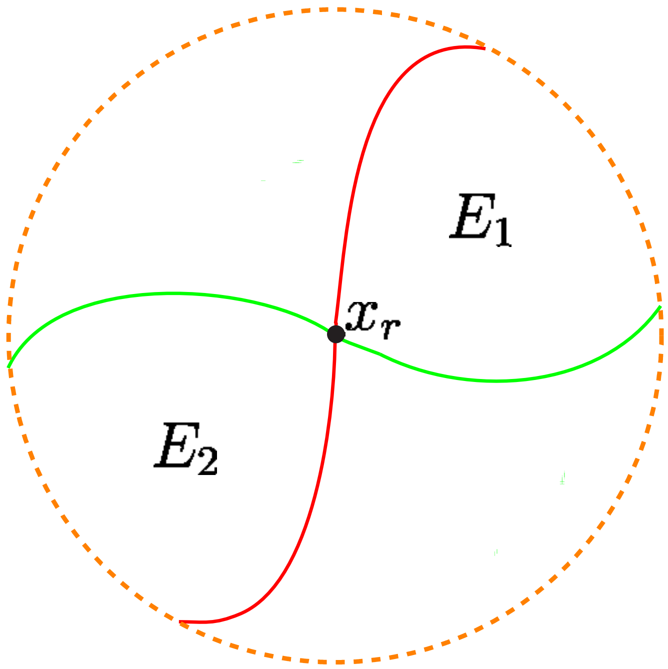

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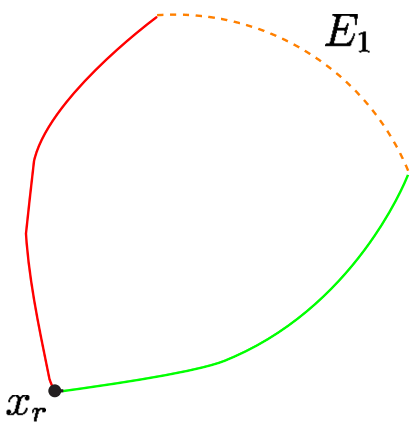

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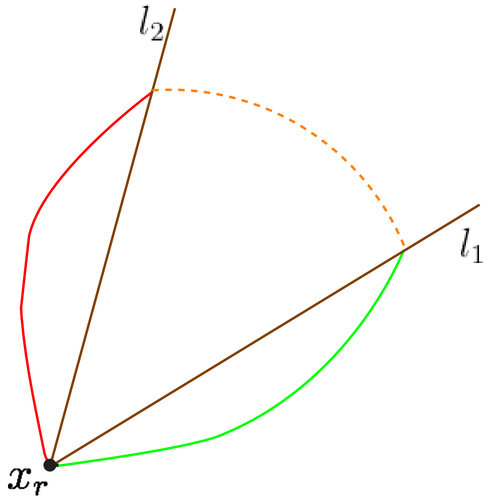

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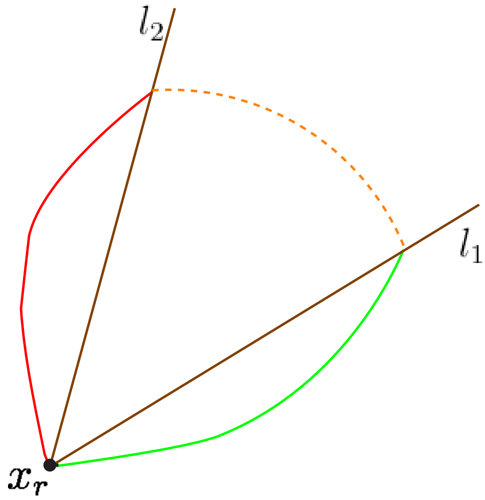

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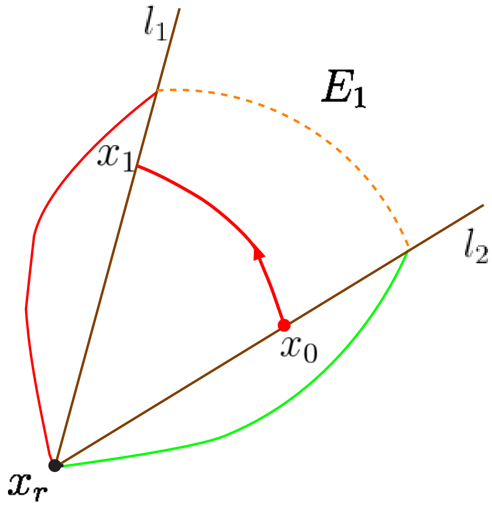

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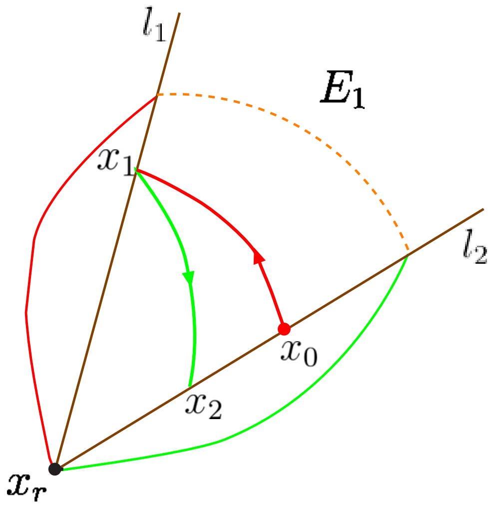

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We suppose that:

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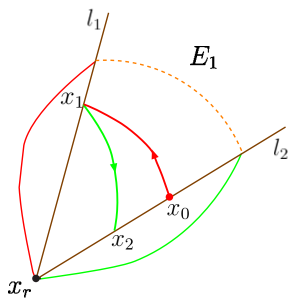

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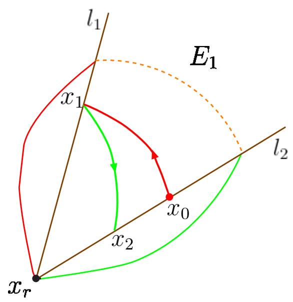

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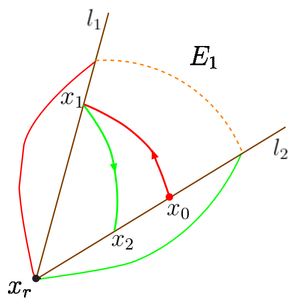

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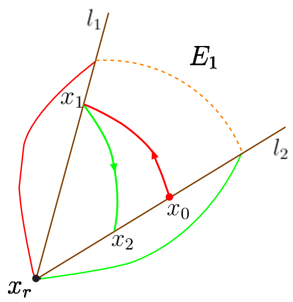

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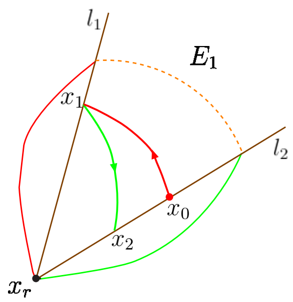
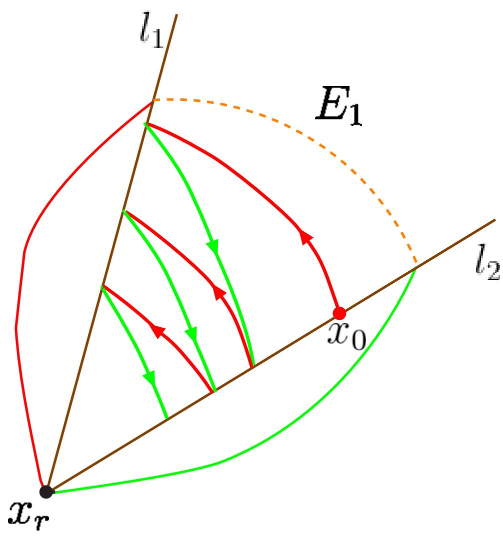


RESULTS ABOUT CONTROL

THEOREM

If $E_1 \cap \{x \in \mathbb{R}^2 : \det(f_1(x), f_2(x) > 0\} \neq \emptyset$, then there exists a cone $\mathrm{C}(x_r; l_1, l_2)$ such that for each initial condition $x_0 \in \mathrm{C}(x_r; l_1, l_2)$ there exists a switching law σ such that the solution of the switched system starting from x_0 converge to x_r.



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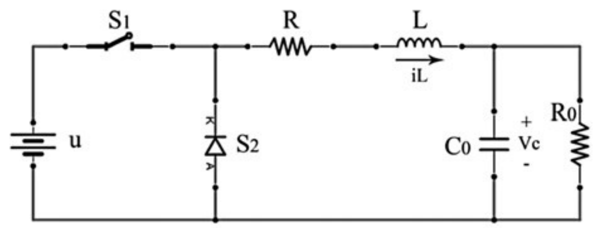
A DC-DC CONVERTER MODEL



- i_L is the inductor current.
- v_c is the capacitor voltage.
- $x = (i, v)$ is the state variable.

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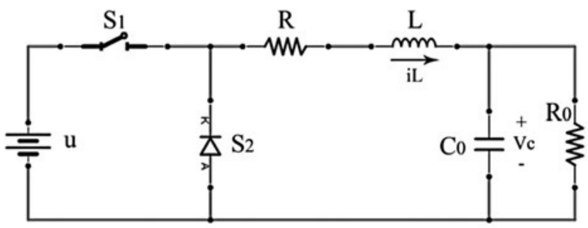

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When S_1 is ON,

$$\dot{x} = \begin{pmatrix} -R/L & -1/L \\ 1/C_0 & -1/(R_0 C_0) \end{pmatrix} x + \begin{pmatrix} 1/L \\ 0 \end{pmatrix} u$$

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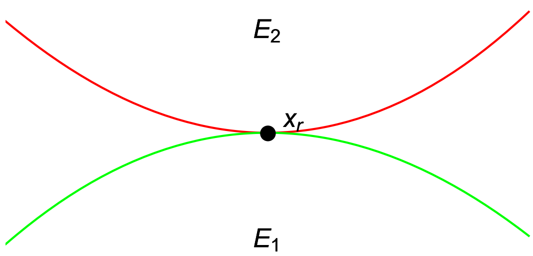

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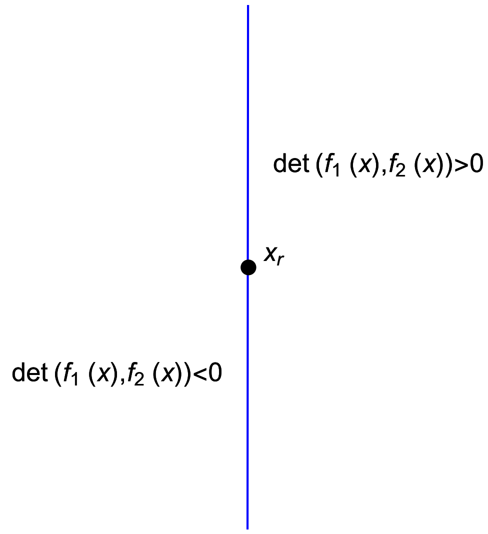

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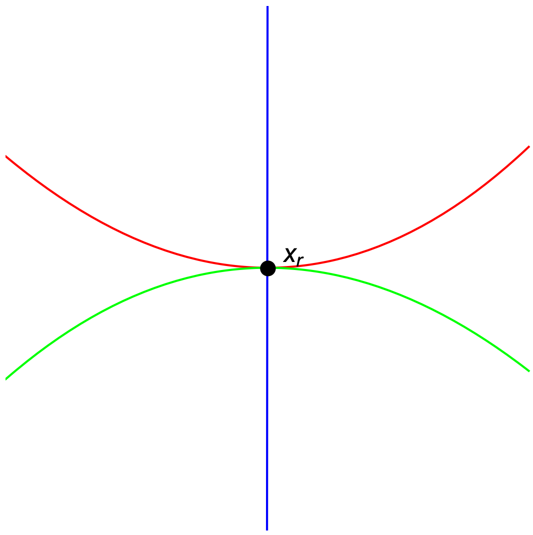

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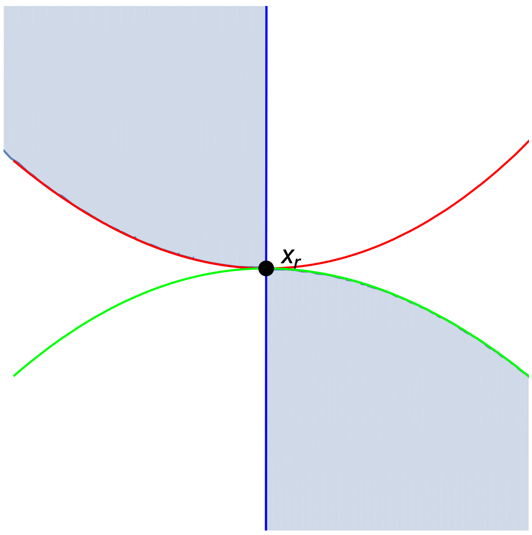

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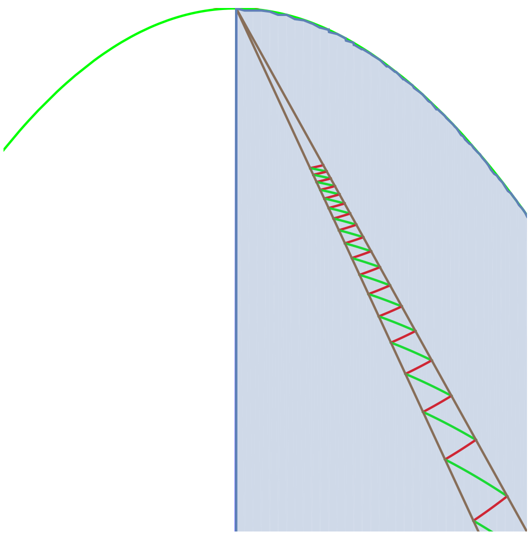

Numerical example

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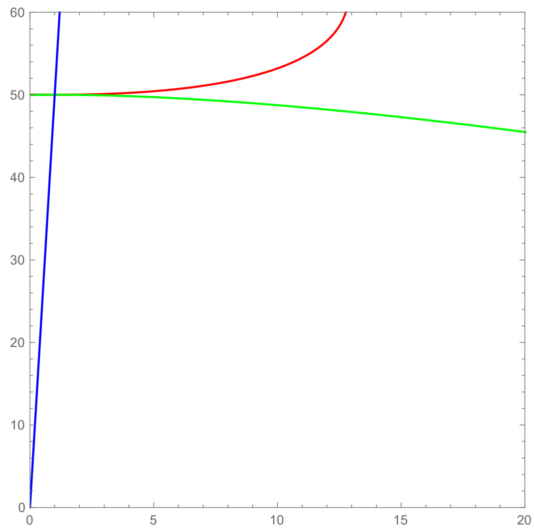

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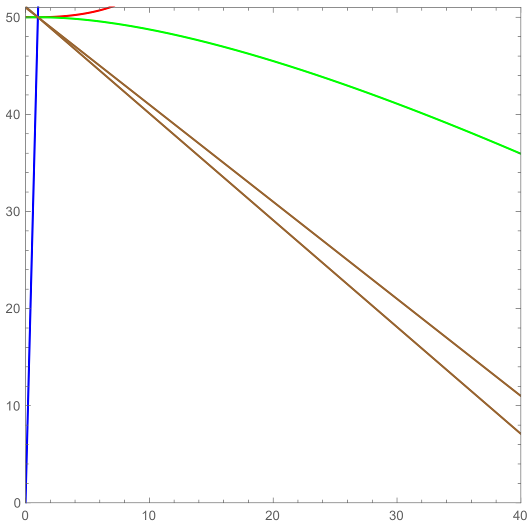

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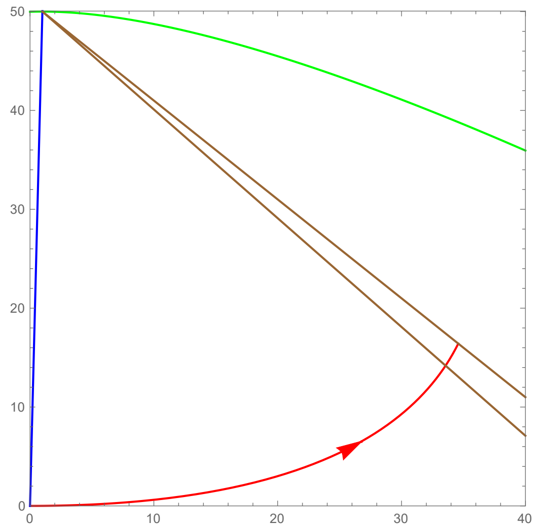

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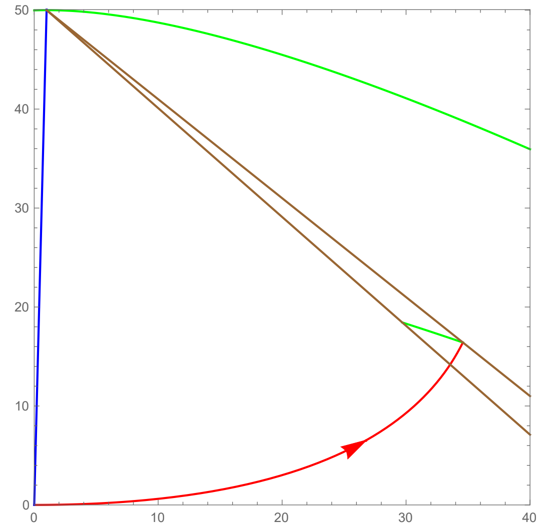

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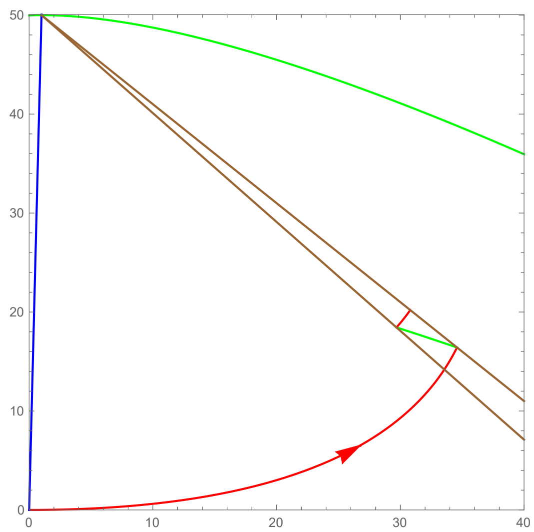

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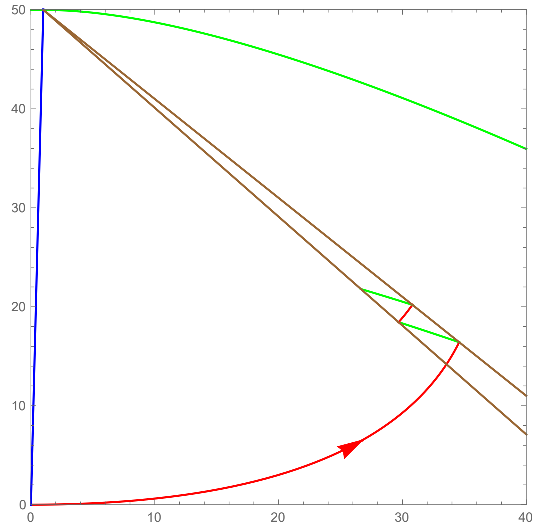

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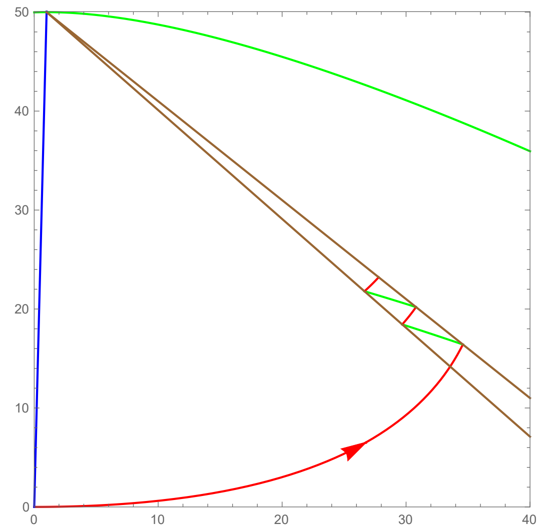

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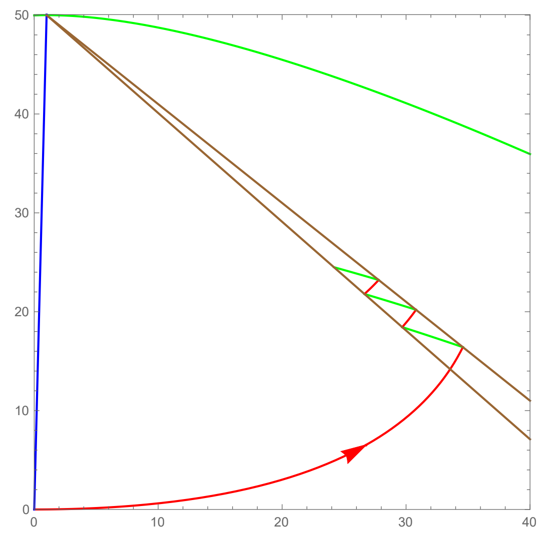

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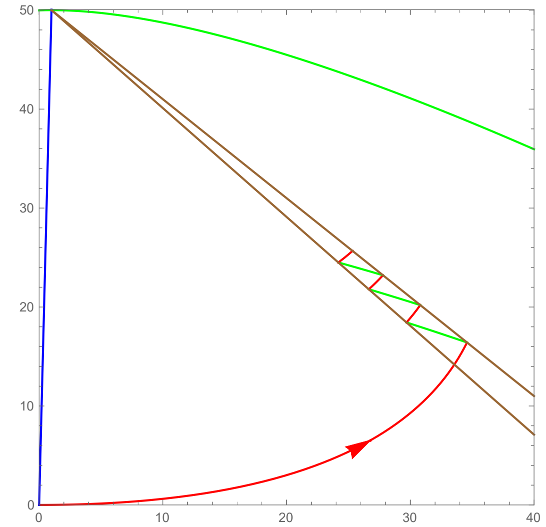
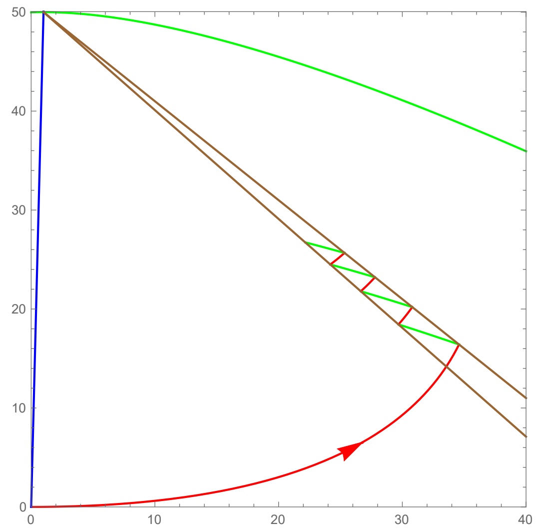


NUMERICAL EXAMPLE



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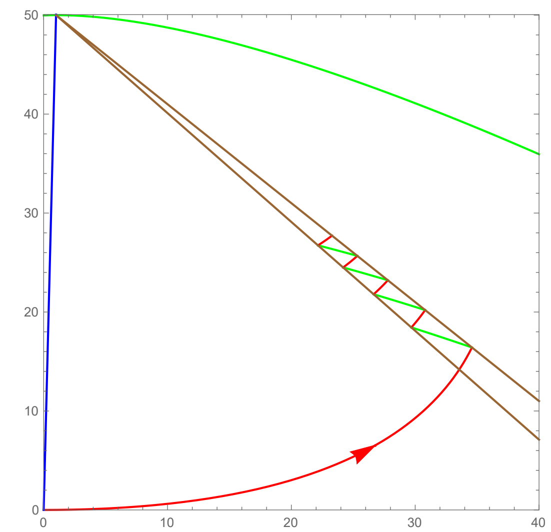

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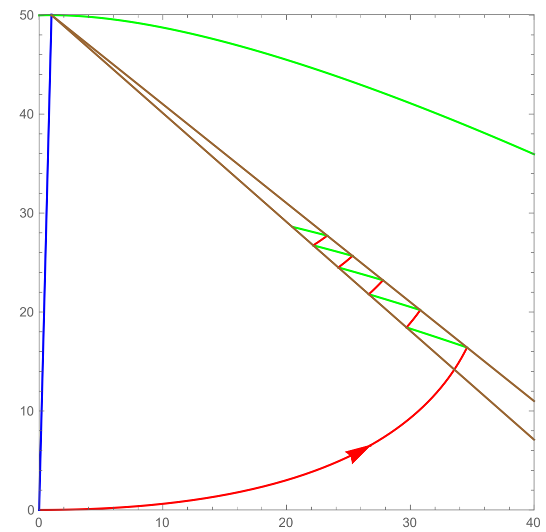

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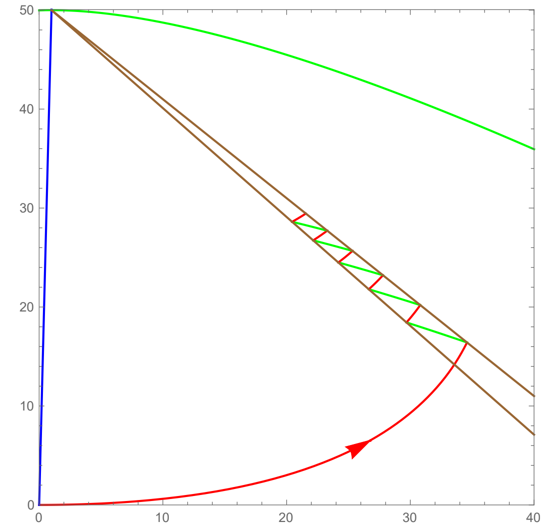

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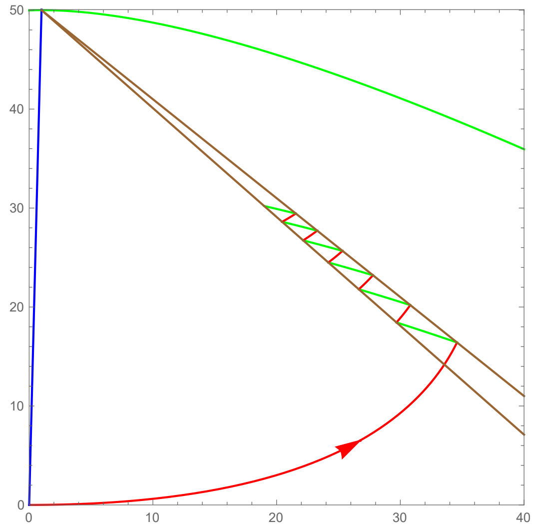

Thank you for your attention!